BAULKHAM HILLS HIGH SCHOOL

2015 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 100
Exam consists of 11 pages.

This paper consists of TWO sections.

Section 1 – Page 2-4 (10 marks)

- Attempt Question 1-10
- Allow about **15** minutes for this section

Section II – Pages 5-10 (90 marks)

- Attempt questions 11-16
- Allow about **2 hours and 45** minutes for this section

Table of Standard Integrals is on page 11

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for questions 1-10

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1. Simplify $\frac{2}{2-\sqrt{3}} - \frac{1}{2+\sqrt{3}}$

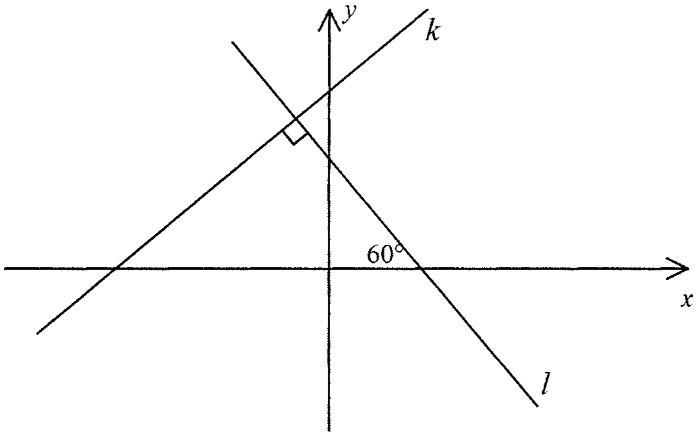
(A) -1

(B) $2+\sqrt{3}$

(C) $\frac{-2-3\sqrt{3}}{5}$

(D) $2+3\sqrt{3}$

2. The diagram shows line l and k. What is the slope of line k?



(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $-\frac{1}{\sqrt{3}}$

3. Which inequality defines the domain of the function $f(x) = \frac{1}{\sqrt{x^2-4}}$

(A) $x < -2$ or $x > 2$
(B) $x < -2$
(C) $-2 < x < 2$
(D) $x > 2$

4. A parabola has a focus (0,6) and its directrix $y = 2$.
What is the equation of the parabola?
(A) $x^2 = -8(y-4)$
(B) $x^2 = -16(y-5)$
(C) $x^2 = 8(y-4)$
(D) $x^2 = 16(y-5)$

5. What is the solution of $3^m = 8$?

(A) $m = \frac{\log_e 8}{3}$

(B) $m = \frac{8}{\log_e 3}$

(C) $m = \log_e \left(\frac{8}{3}\right)$

(D) $m = \frac{\log_e 8}{\log_e 3}$

6. What is the derivative of $\frac{e^{-x}}{x}$?

(A) $\frac{-xe^{-x} - e^{-x}}{x^2}$

(B) $\frac{-xe^{-x} + e^{-x}}{x^2}$

(C) $\frac{e^{-x} + xe^{-x}}{x^2}$

(D) $\frac{e^{-x} - xe^{-x}}{x^2}$

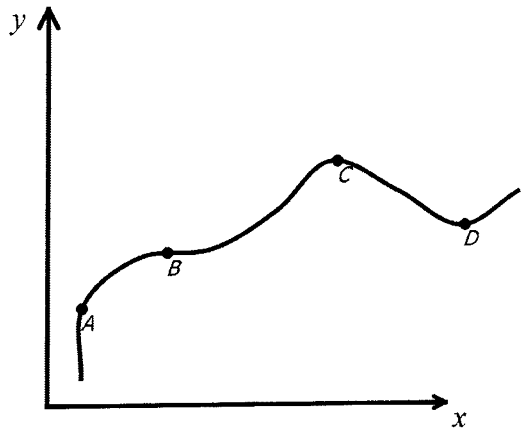
7. The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β.
The value of $\alpha\beta + (\alpha^2 + \beta^2)$ is

(A) -10
(B) -8
(C) 8
(D) 10

8. The value of $\int_1^4 \frac{2}{x}\,dx$ is

(A) $\frac{\ln 4}{2}$
(B) $\ln 4$
(C) $2\ln 3$
(D) $4\ln 16$

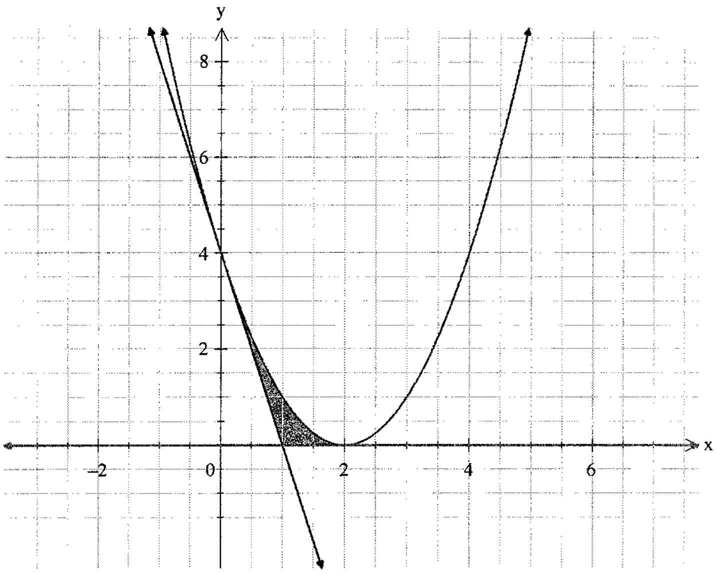
9. A point on the graph of $y = f(x)$ has $f'(x) = 0$ *and* $f''(x) = 0$.



The point is

(A) A
(B) B
(C) C
(D) D

10. Which of the following sets of inequalities describes the shaded region in the diagram?:



(A) $y \le (x-2)^2,\ y \ge 0$ and $y \ge 4 - 4x$
(B) $y \ge (x-2)^2,\ y \ge 0$ and $y \ge 4 + 4x$
(C) $y \le (x+2)^2,\ y \le 0$ and $y \ge 4 - 4x$
(D) $y \le (x+2)^2,\ y \ge 0$ and $y \ge 4 - 4x$

END OF SECTION I

Section II

90 marks

Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section
Answer each question in the appropriate page in the writing booklet.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start on the appropriate page in the answer booklet

(a) Evaluate $\frac{3.42^2-1.2^2}{\sqrt{36+1.2}}$ correct to 3 significant figures. **2**

(b) Rationalise the denominator of $\frac{5}{3-\sqrt{6}}$ **2**

(c) Solve $\frac{1}{2}(x-2) = \frac{1}{8}(1-3x)+4$ **2**

(d) If $\tan\theta = \frac{5}{7}$ and $\sin\theta < 0$ find the exact value of $\sec\theta$ **2**

(e) Solve $|15-4m| \leq 3$ **2**

(f) Find the sum of the first 15 terms of the series **3**

$1+3+3^2+3^3+\cdots$

(g) Solve $2\sin\theta + 1 = 0$ for $0 \leq \theta \leq 2\pi$ **2**

Question 12 (15 marks) Start on the appropriate page in the answer booklet

(a) Differentiate with respect to x

(i) $(2x^2 + 1)^6$ **2**

(ii) $x^3 \ln x$ **2**

(iii) $\frac{\sin x}{e^x}$ **2**

(b) $\int \cos 2x + e^{5x}\, dx$ **2**

(c) Evaluate $\int_0^{\pi} \sin\theta + 1 \; d\theta$ **2**

(d) Find the equation of the normal to the curve

$y = x^3 - 2x - 1$ at the point where $x = 2$ **2**

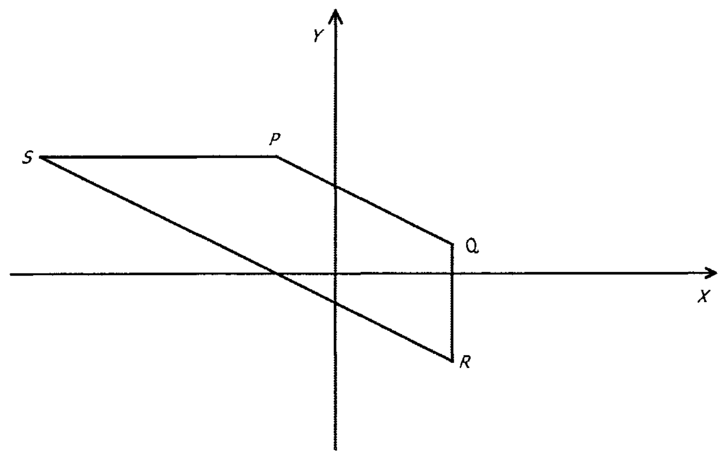
(e) (i) For the function $y = \log_{10} x$, copy and complete the table to 3 decimal places in your exam booklet. **1**

x	1	2	3	4	5
y	0	0.301	0.477		

(ii) Apply the trapezoidal rule with 4 subintervals to find an approximation to two decimal places of $\int_1^5 \log_{10} x \; dx$ **2**

Question 13 (15 marks) Start on the appropriate page in the answer booklet

(a) In the quadrilateral *PQRS* the coordinates of the points *P* and *Q* are (-2,4) and (4,1) respectively. The equation of line *SR* is $x + 2y + 2 = 0$.



NOT TO SCALE

(i) Find the gradients of *PQ* and *RS*. Hence, explain why the quadrilateral *PQRS* is a trapezium? **2**

(ii) Find the length of *PQ* in exact form. **2**

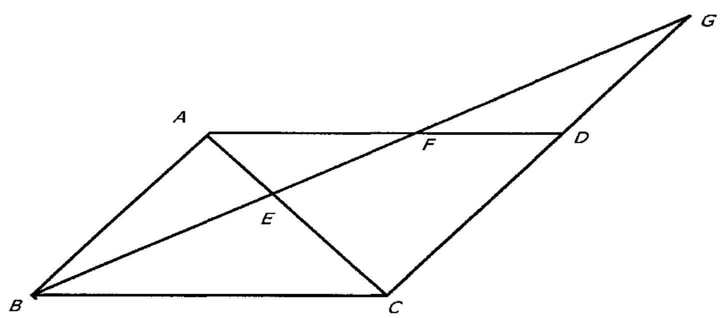
(iii) The line *QR* is parallel to the y axis, find the coordinates of point *R*. **1**

(iv) Find the perpendicular distance from *P* to the line *RS*. **2**

(v) If the length of *RS* is $\sqrt{85}$ units find the area of trapezium *PQRS*. **2**

(b) An infinite geometric series has a limiting sum of 3. If the first term of the series is equal to the common ratio, find the first term of this series. **2**

(c) ABCD is a parallelogram. A straight line through B intersects diagonal AC at E, side AD at F and side CD extended to G. BE and EF are 24 and 18 respectively.



(i) Prove Δ AEF /// ΔBCE **2**

(ii) Hence or otherwise find FG. **2**

Question 14 (15 marks) Start on the appropriate page in the answer booklet

a) Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$, find the values of

(i) $\log_7 0.4$ **1**

(ii) $\log_7 50$ **2**

(b) A function $y = f(x)$ is defined by the following features:

$f''(x) > 0$ for $x < -1$ and $1 < x < 3$

$f'(x) = 0$ when $x = -3, 1$ and 5

$f(x) = 0$ when $x = 1$

(i) Identify the x values of the stationary points and determine the nature of each point. **2**

(ii) Sketch a possible graph of the function. **1**

(c) Karen contributes to a superannuation fund. She contributes \$250 at the start of every quarter. The investment pays 8% pa interest, compounding quarterly. She continues making contributions for 30 years.

(i) How much does she contribute altogether? **1**

(ii) What is the value of her initial \$250 investment at the end of 30 years? **1**

(iii) Find the total value of her superannuation. **3**

(d) (i) Show that $\dfrac{d}{dx}(x\ln x - x) = \ln x$ **1**

(ii) Hence evaluate $\displaystyle\int_1^e \ln x \, dx$ **3**

Question 15 (15 marks) Start on the appropriate page in the answer booklet

(a) The area bounded by the curve $y = \sqrt{\frac{2x}{3x^2-1}}$ between the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Find the exact volume of the solid of revolution formed. **3**

(b) Rita has just been shopping and purchased 3 cans of baked beans and 2 cans of spaghetti. While she is on the phone her little brother removes all the labels from all the cans so that they all look alike now. Her brother wants baked beans for lunch. Rita decides to open only two cans. She selects the two cans at random.

(i) Draw a probability tree to illustrate the situation. **2**

(ii) What is the probability that Rita selects two cans of baked beans? **1**

(iii) What is the probability that she selects exactly one can of baked beans? **2**

(iv) Rita opens one can and discovers that it is spaghetti.
What is the probability that the other can is baked beans? **2**

(c) The velocity of an object is given by the equation $v = 6t - 8 - t^2$ where the time(t) is in seconds and the velocity (v) is in m/s. Initially, the object is 5m to the right of the origin.

(i) Find an equation for the displacement of the object. **2**

(ii) When is the object momentarily at rest? **1**

(iii) Find the distance travelled by the object in the first four seconds. **2**

Question 16 (15 marks) Start on the appropriate page in the answer booklet

(a) (i) Show that $\sin\theta \cot\theta = \cos\theta$ **1**

(ii) Hence solve $27\cos\theta \sin\theta \cot\theta = \sec\theta$ where $0 \leq \theta \leq 2\pi$ **2**

(b) The percentage of Carbon in an organism falls exponentially after the death of the organism. After 1845 years 80% of the original concentration of Carbon remains. Use the equation $C = C_o e^{-kt}$ to represent the exponential fall of Carbon.

(i) Find the exact value of k. **2**

(ii) An artwork containing this organism has 65% of the original concentration of Carbon.
How long has this organism been dead? Give the answer to the nearest year. **2**

(iii) A sea sponge containing this organism has been dead for 12000 years. What percentage (to 1 decimal place) of the original Carbon concentration does it have? **2**

(c) Two sailors are paid to bring a motor boat back to a harbour from an island, a total distance of 1200 km. They are each paid \$25 per hour for the time spent at sea. The boat uses fuel at a rate of $20 + \frac{v^2}{10}$ litres per hour where the speed of the boat is v km per hour. Diesel fuel costs \$1.25 per litre.

(i) Show that to bring the boat back from the island, the total cost (\$C) to the owner is $C = \frac{90000}{v} + 150v$ **3**

(ii) Find the speed that minimises the cost and determine this cost, giving your answer to the nearest dollar. **3**

End of Examination

Mathematics Advanced – 2015 Trial Solutions.

PART A

1. D
2. C
3. A
4. C
5. D
6. A
7. D
8. B
9. B
10. A

Mathematics Advanced – 2015 TRIAL solutions.

Question 11.

a) $1.68160\ldots$
$= 1.68$

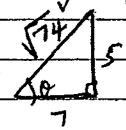
3 sig ✓
correct ans ✓

b) $\dfrac{5(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})}$ ✓

$= \dfrac{15+5\sqrt{6}}{3}$ ✓

c) $4(x-2) = (1-3x)+32$ ✓
$4x-8 = 1-3x+32$
$7x = 41$
$x = \dfrac{41}{7}$ ✓

d) $\tan\theta = \dfrac{5}{7}$ and $\sin\theta < 0$ ✓



$\cos\theta = -\dfrac{7}{\sqrt{74}}$

$\sec\theta = -\dfrac{\sqrt{74}}{7}$ ✓

e) $|15-4m| \leq 3$

$15-4m \leq 3$ or $-15+4m \leq 3$

$m \geq 3$ ✓ $\quad m \leq 18/4$

$m \leq 9/2$ ✓

f) $a=1$, $r=3$, $n=15$ ✓ $\quad S_n = \dfrac{a(r^n-1)}{r-1}$

$S_{15} = \dfrac{1(3^{15}-1)}{2} = 7174453$ ✓

g) $\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}(-\frac{1}{2})$

$= \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓ ✓

Question 12

a)(i) $y = (2x^2+1)^6$

$\frac{dy}{dx} = 6(2x^2+1)^5 \times 4x$ ✓ ✓

$= 24x(2x^2+1)^5$

(ii) $y = x^3 \ln x$

$\frac{dy}{dx} = x^3 \cdot \frac{1}{x} + \ln x \times 3x^2$ ✓ ✓

$= x^2(1+3\ln x)$

(iii) $y = \frac{\sin x}{e^x}$

$\frac{dy}{dx} = \frac{e^x \cdot \cos x - \sin x \times e^x}{(e^x)^2}$ ✓

$= \frac{\cos x - \sin x}{e^x}$ ✓

b) $\int (\cos 2x + e^{5x})\,dx = \frac{1}{2}\sin 2x + \frac{1}{5}e^{5x} + C$ ✓ ✓

c) $\int_0^{\pi} (\sin\theta + 1)\,d\theta = \left[-\cos\theta + \theta\right]_0^{\pi} = (1+\pi) - (-1)$ ✓

$= 2 + \pi$ ✓

d) $\frac{dy}{dx} = 3x^2 - 2$

when $x = 2$ $\frac{dy}{dx} = 10$

$\therefore$ gradient of normal at $x = 2$, $m = -\frac{1}{10}$ ✓

when $x = 2$, $y = 8-4-1 = 3$ $\therefore (2,3)$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{10}(x-2)$ ✓

$x + 10y - 32 = 0$

e)(i) $y = \log_{10} x$

x	1	2	3	4	5
y	0	0.301	0.477	0.602	0.699

✓

(ii) $\int_1^5 \log_{10} x\,dx = \frac{h}{2}\left[f(1) + f(5) + 2\left(f(2) + f(3) + f(4)\right)\right]$

$= \frac{1}{2}\left[0 + 0.699 + 2(0.301 + 0.477 + 0.602)\right]$ ✓

$= 1.7295$

$\simeq 1.73$ (2dp) ✓

Question 13

a)(i) $m_{PQ} = \frac{4-1}{-2-4} = -\frac{1}{2}$; $m_{RS} = -\frac{1}{2}$ ✓

a pair of opposite sides are $\parallel$ ✓

(ii) $d_{PQ} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(-2-4)^2 + (4-1)^2}$ ✓ $= \sqrt{45}$

$= 3\sqrt{5}$ ✓

(iii) $R(4,a)$ this lies in $x + 2y + 2 = 0$

$4 + 2a + 2 = 0 \Rightarrow a = -3$

$\therefore R(4,-3)$ ✓

(iv) $P(-2,4)$; $x+2y+2=0$

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|-2+8+2|}{\sqrt{5}} \checkmark = \frac{8}{\sqrt{5}} \checkmark$$

(v) Area of a trapezium $= \frac{1}{2}h(a+b)$.

$$= \frac{1}{2}\cdot\frac{8}{\sqrt{5}}\left(\sqrt{85}+\sqrt{45}\right) \checkmark$$

$$= 12 + 4\sqrt{17} \text{ or } 28.5 \text{ (1dp)} \checkmark$$

b) $S_\alpha = 3$, a = common ratio $= r$ ✓

$$S_\alpha = \frac{a}{1-r}$$

$$3 = \frac{a}{1-a} \Longrightarrow 3-3a = a$$

$$a = 3/4 \checkmark$$

c) (i) In $\triangle AEF$ and $\triangle BCE$ — all correct statements ✓

$\angle AEF = \angle BEC$ [vertically opposite ∠] — all correct reasons ✓

$\angle EBC = \angle EFA$ [alternate ∠ on || lines, AF||BC]

$\therefore \triangle AEF ||| \triangle BCE$ [2 pairs of matching ∠ are =]

(ii) $\frac{BC}{AF} = \frac{24}{18}$; ✓ —① $\frac{42}{FG} = \frac{AF}{FD}$

$$\therefore \frac{42}{42+FG} = \frac{AF}{AF+FD}$$

$$= \frac{AF}{BC} \quad \text{—②}$$

① & ② $\Longrightarrow \frac{24}{18} = \frac{42+FG}{42}$

$\therefore FG = 14$ ✓

Question 14

a)(i) $\log_7(0.4) = \log_7(2/5)$

$= 0.36 - 0.83$

$= -0.47$ ✓

(ii) $\log_7(50) = \log_7(2\times5^2)$

$= \log_7 2 + 2\log_7 5$ ✓

$= 2.02$ ✓

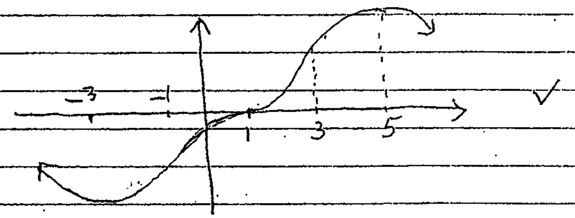
b) (i) as $f'(x)=0$, $-3, 1$ and 5 are x values of stationary points

$f''(x) > 0$ for $x < -1$ $\Longrightarrow$ Concave up

$f''(x) > 0$ for $1 < x < 3$ $\Longrightarrow$ Concave up — all ✓

$\therefore$ at $x=-3$ a minimum turning point

$x=5$ a maximum turning point



c) @ 2% per quarter ; terms 30 years = 120 quarters.

(i) $250 \times 120 = \$30{,}000$ ✓

(ii) $A_n = P\left(1+\frac{r}{100}\right)^n$

$$= 250\left(1+\frac{2}{100}\right)^{120}$$

$= \$2691.29$ ✓

) 1st instalment is worth $= 250(1\cdot02)^{120}$

2nd $= 250(1\cdot02)^{119}$

⋮

last (120th instalment) $= 250\times(1\cdot02)^{1}$

Superannuation $= 250\times1\cdot02 + 250\times1\cdot02^{2} + \cdots + 250\times1\cdot02^{119} + 250\times1\cdot02^{120}$ ✓

This is a GP for which $S_n = \dfrac{a(r^n-1)}{r-1}$

$\therefore S_{120} = 250\times1\cdot02\times\dfrac{(1\cdot02^{120}-1)}{1\cdot02-1}$ ✓

$= \$124505\cdot83$ ✓

d (i) $\dfrac{d}{dx}(x\ln x - x)$

$= x\times\dfrac{1}{x} + \ln x\times 1 - 1$ ✓

$= \ln x$

(ii) $\displaystyle\int_1^e \ln x\,dx = \Big[x\ln x - x\Big]_1^e$ ✓

$= \Big[e\times1 - e\Big] - \Big[1\times0 - 1\Big]$ ✓

$= 1$ ✓

Question 15

a) $V = \pi\displaystyle\int_a^b y^2\,dx$ where $y = \sqrt{\dfrac{2x}{3x^2-1}}$, $a=1$, $b=3$ ✓

or

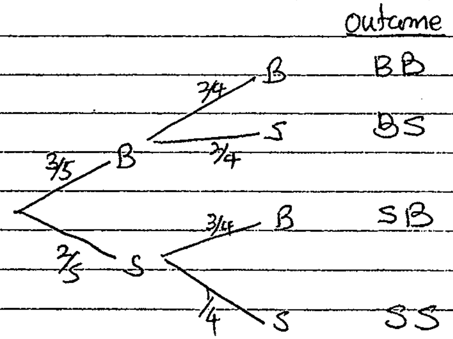
$= \pi\displaystyle\int_1^3 \dfrac{2x}{3x^2-1}\,dx$ ✓

$= \dfrac{\pi}{3}\displaystyle\int_1^3 \dfrac{6x}{3x^2-1}\,dx$

$= \dfrac{\pi}{3}\Big[\ln(3x^2-1)\Big]_1^3$ ✓

$= \dfrac{\pi}{3}\ln 13.$ ✓

b) i)



Correct outcome ✓
Correct prob. ✓

(ii) $P(\text{Select } 2B) = \dfrac{3}{5}\times\dfrac{2}{4} = \dfrac{3}{10}$ ✓

(iii) $P(\text{exactly } 1B) = P(BS) + P(SB)$ ✓

$= \dfrac{3}{10} + \dfrac{3}{10}$

$= \dfrac{3}{5}$ ✓

(iv) $\dfrac{3}{4}$ ✓✓

c) (i) $\frac{dx}{dt} = 6t - 8 - t^2$

$\therefore x = \frac{6t^2}{2} - 8t - \frac{t^3}{3} + C$ ✓

when $t = 0$, $x = 5$ $\therefore C = 5$

$x = 3t^2 - 8t - \frac{t^3}{3} + 5$ ✓

(ii) $v = 0$ for particle to be at rest

$t^2 - 6t + 8 = 0$

$\Longrightarrow t = 2 ; t = 4.$

i.e at 2nd and 4th seconds the particle will be at rest. ✓

(iii) $d = \left| \int_0^2 (6t - 8 - t^2)\, dt \right| + \left| \int_2^4 (6t - 8 - t^2)\, dt \right|$ ✓

or alternative method.

$= 8\text{m}.$ ✓

Question 16

a) (i) $LHS = \sin\theta . \cot\theta$

$= \sin\theta . \frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

$= RHS$ ✓

$\therefore \sin\theta \cot\theta = \cos\theta.$

(ii) $27\cos\theta . \sin\theta \cot\theta = \sec\theta$

$27\cos^2\theta = \frac{1}{\cos\theta}$

$\cos^3\theta = \frac{1}{27}$ ✓ $\Longrightarrow \cos\theta = \frac{1}{3}$

$\theta = 70°32', 289°28'$ ✓

b (i) $C = C_0 e^{-kt}$

$t = 1845$, $C = \frac{80}{100} C_0$ ✓

$\therefore \frac{80}{100} C_0 = C_0 e^{-1845k}$

$-1845k = \ln(0.8)$

$k = -\frac{1}{1845} \ln(0.8)$

$= 1.209 \times 10^{-4}$ ✓ —— ①.

(ii) $C = \frac{65}{100} C_0$ $t = ?$

$\frac{65}{100} C_0 = C_0 e^{-kt}$ ✓

$-kt = \ln(0.65)$

$t = -\frac{1}{k} \ln(0.65)$

using ①

$t = 3561.80 \ldots$

$\simeq 3562$ years. ✓

(iii) $C = C_0 e^{-kt}$

$\frac{C}{C_0} = e^{-kt}$

$k = 1.209 \times 10^{-4}$ $t = 12000 \Longrightarrow \frac{C}{C_0} = 0.2343$ ✓

$\therefore$ % concentration $= 23.43\%$ ✓

c (i) distance travelled $= 1200$km

time spent $= \frac{1200}{v}$ h

labour cost $= \$ \frac{1200}{v} \times 25 \times 2$ ✓ —— ①

Fuel consumed $= \left(20 + \frac{v^2}{10}\right) \frac{1200}{v}$ L

Fuel Cost $= \$ \left(20 + \frac{v^2}{10}\right) \frac{1200}{v} \times 1.25$ —— ② ✓

① & ② $\Longrightarrow \therefore$ Total Cost $C = \frac{1200}{v} \times 50 + \frac{1200}{v} \times 1.25 \left(20 + \frac{v^2}{10}\right)$

$= \frac{90000}{v} + 150v$

$C = \frac{90000}{V} + 150V$

C is minimum when $\frac{dc}{dv} = 0$ and $\frac{d^2c}{dv^2} > 0$ ✓

$\frac{dc}{dv} = -90000V^{-2} + 150V$

$\frac{d^2c}{dv^2} = 180000V^{-3} + 0$

$= \frac{180000}{V^3}$ [positive $\forall\, V > 0$

$\frac{dc}{dv} = 0 \implies \frac{90000}{V^2} = 150$

$V^2 = 600$

$V = 10\sqrt{6}$ ✓

$\therefore$ when $V = 10\sqrt{6}$ cost is minimised.

$C_{min} = \frac{90000}{10\sqrt{6}} + 150 \times 10\sqrt{6}$

$\simeq \$7348$ ✓